5.1

TABLE Solution $X(\lambda_n, x)$, the Norm $N(\lambda_n)$, and the Eigenvalues λ_n of the Differential Equation

$$\frac{d^2X(x)}{dx^2} + \lambda^2 X(x) = 0 \quad \text{in} \quad 0 < x < L$$

Case No.	Boundary Condition at $x = 0$	Boundary Condition at $x = L$	$X(\lambda_n, x)$	$1/N(\lambda_n)$	Eigenvalues λ_n Are Positive Roots of
1	$-\frac{dX}{dx} + H_1 X = 0$	$\frac{dX}{dx} + H_2 X = 0$	$\lambda_n \cos\lambda_n x + H_1 \sin\lambda_n x$	$2\left[(\lambda_n^2 + H_1^2)\left(L + \frac{H}{\lambda_n^2 + H_2^2}\right) + H_1\right]^{-1}$	$\tan\lambda_n L = \frac{\lambda_n (H_1 + H_2)}{\lambda_n^2 - H_1 H_2}$
2	$-\frac{dX}{dx} + H_1 X = 0$	$\frac{dX}{dx} = 0$	$\cos\lambda_n (L - x)$	$2\frac{\lambda_n^2 + H_1^2}{L(\lambda_n^2 + H_1^2) + H_1}$	$\lambda_n \tan\lambda_n L = H_1$
3	$-\frac{dX}{dx} + H_1 X = 0$	$X = 0$	$\sin\lambda_n (L - x)$	$2\frac{\lambda_n^2 + H_1^2}{L(\lambda_n^2 + H_1^2) + H_1}$	$\lambda_n \cot\lambda_n L = -H_1$
4	$\frac{dX}{dx} = 0$	$\frac{dX}{dx} + H_2 X = 0$	$\cos\lambda_n x$	$2\frac{\lambda_n^2 + H_2^2}{L(\lambda_n^2 + H_2^2) + H_2}$	$\lambda_n \tan\lambda_n L = H_2$
5	$\frac{dX}{dx} = 0$	$\frac{dX}{dx} = 0$	$\cos\lambda_n x$[a]	$\frac{2}{L}$ for $\lambda_n \neq 0$; $\frac{1}{L}$ for $\lambda_0 = 0$[a]	$\sin\lambda_n L = 0$[a]
6	$\frac{dX}{dx} = 0$	$X = 0$	$\cos\lambda_n x$	$\frac{2}{L}$	$\cos\lambda_n L = 0$
7	$X = 0$	$\frac{dX}{dx} + H_2 X = 0$	$\sin\lambda_n x$	$2\frac{\lambda_n^2 + H_2^2}{L(\lambda_n^2 + H_2^2) + H_2}$	$\lambda_n \cot\lambda_n L = -H_2$
8	$X = 0$	$\frac{dX}{dx} = 0$	$\sin\lambda_n x$	$\frac{2}{L}$	$\cos\lambda_n L = 0$
9	$X = 0$	$X = 0$	$\sin\lambda_n x$	$\frac{2}{L}$	$\sin\lambda_n L = 0$

[a] For this particular case $\lambda_0 = 0$ is also an eigenvalue corresponding to $X = 1$. $H_1 = h_1/k$ and $H_2 = h_2/k$.