```
> restart;
>
> with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(1)**

```
> # Example 1: Consider the following Sturm -Liouville eigenvalue problem
> #          ϕ''(x)+λ^2ϕ(x) = 0
> #        ϕ'(0)=0
> #       ϕ(1)=0
> # Determine the eigenvalues and Eigen functions
>
>
> de := u"(x) + λ^2·u(x) = 0;
```

$$de := \frac{d^2}{dx^2} u(x) + \lambda^2 u(x) = 0 \qquad \textbf{(2)}$$

```
> sol := rhs(dsolve(de, u(x)));
```

$$sol := \_C1 \sin(\lambda x) + \_C2 \cos(\lambda x) \qquad \textbf{(3)}$$

```
> BC1 := subs(x=0, diff( sol, x));
```

$$BC1 := \_C1\, \lambda \cos(0) - \_C2\, \lambda \sin(0) \qquad \textbf{(4)}$$

```
> BC2 := subs(x=1, sol);
```

$$BC2 := \_C1\,\sin(\lambda) + \_C2\,\cos(\lambda) \qquad \textbf{(5)}$$

> $AA := GenerateMatrix(\{BC1, BC2\}, \{\_C1, \_C2\});$

$$AA := \begin{bmatrix} \lambda & 0 \\ \sin(\lambda) & \cos(\lambda) \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \textbf{(6)}$$

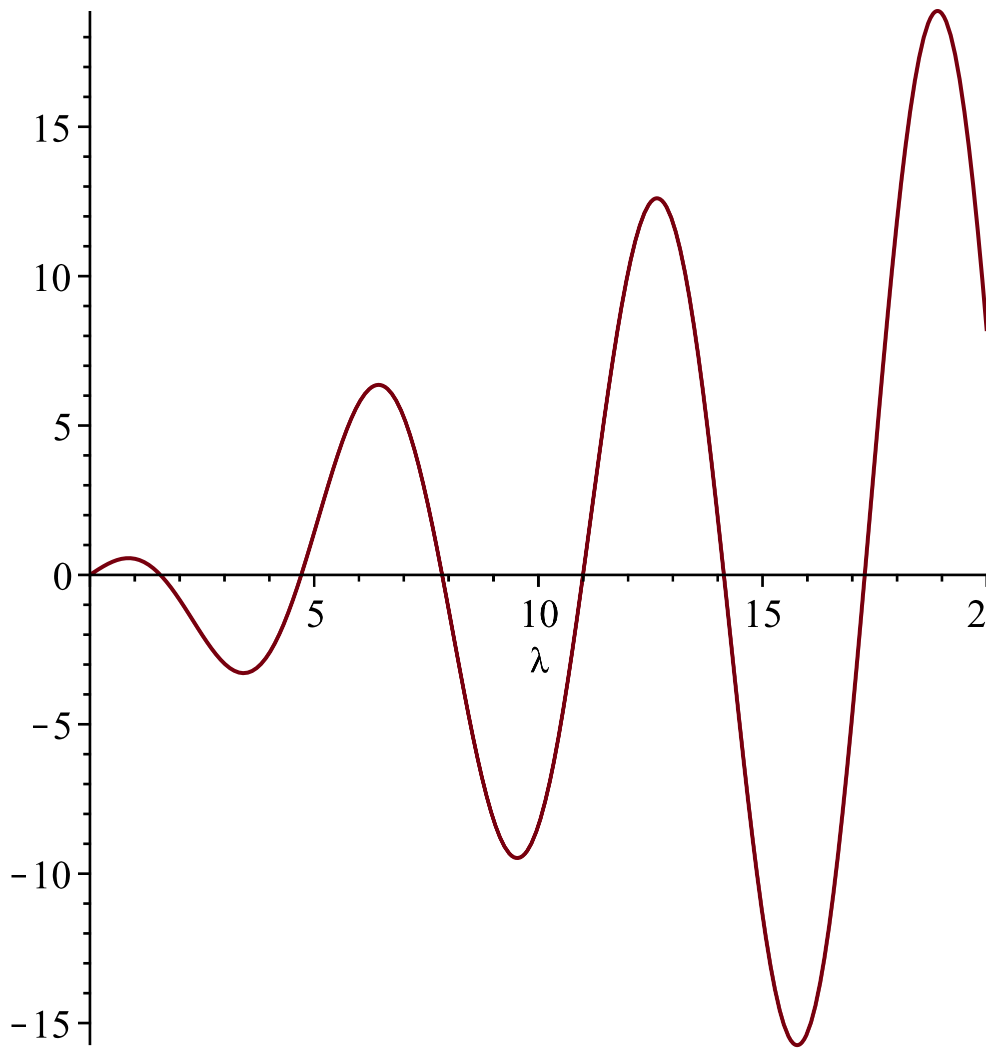
> $A := AA[1];$

$$A := \begin{bmatrix} \lambda & 0 \\ \sin(\lambda) & \cos(\lambda) \end{bmatrix} \qquad \textbf{(7)}$$

> $p := Determinant(A);$

$$p := \lambda\cos(\lambda) \qquad \textbf{(8)}$$

> $plot(p, \lambda = 0..20);$



> $\lambda[1] := fsolve(p, \lambda = 1..2);$

$$\lambda_1 := 1.570796327 \qquad \textbf{(9)}$$

> $\lambda[2] := fsolve(p, \lambda = 4..5);$

$$\lambda_2 := 4.712388980 \qquad \textbf{(10)}$$

> $\lambda[3] := fsolve(p, \lambda = 7..8);$

$$\lambda_3 := 7.853981634 \quad \textbf{(11)}$$

> $\lambda[4] := fsolve(p, \lambda = 10..11);$

$$\lambda_4 := 10.99557429 \quad \textbf{(12)}$$

> $\lambda[5] := fsolve(p, \lambda = 14..15);$

$$\lambda_5 := 14.13716694 \quad \textbf{(13)}$$

>

> *# The eigen functions are*

> $\phi(n) := \cos(\lambda[n] \cdot x);$

$$\phi := n \rightarrow \cos(\lambda_n x) \quad \textbf{(14)}$$

> $seq(\phi(n), n = 1..5);$

$\cos(1.570796327\, x), \cos(4.712388980\, x), \cos(7.853981634\, x), \cos(10.99557429\, x),$ **(15)**
$\cos(14.13716694\, x)$

>

>