Table 4.1

SOLUTION OF $\frac{d}{dx}\left(x^{\alpha}\frac{dy}{dx}\right) + \gamma^2 x^{\beta} y = 0$

Case (i): $\beta - \alpha + 2 \neq 0$. The general solution is

$$y(x) = x^{\nu/\mu} Z_{\nu}(|\gamma|\mu x^{1/\mu}),$$

where

$$\nu = (1-\alpha)/(\beta-\alpha+2), \qquad \mu = 2/(\beta-\alpha+2), \qquad \nu/\mu = (1-\alpha)/2;$$

two particular solutions, corresponding to Z_{ν} and to be selected according to γ and ν, are shown below.

γ	ν	Particular solutions	
Real	Fractional	J_{ν}	$J_{-\nu}$ (or Y_{ν})
	Zero or integer	J_n	Y_n
Imaginary	Fractional	I_{ν}	$I_{-\nu}$ (or K_{ν})
	Zero or integer	I_n	K_n

Case (ii): $\beta - \alpha + 2 = 0$. The general solution is

$$y(x) = x^r;$$

two particular solutions, to be determined according to the roots of

$$r^2 + (\alpha - 1)r + \gamma^2 = 0,$$

are shown below.

$(\alpha-1)^2 - 4\gamma^2$	Particular solutions	
Positive	x^{r_1}	x^{r_2}
Zero	x^{δ}	$x^{\delta}\ln x$
Negative	$x^{\delta}\cos(\epsilon \ln x)$	$x^{\delta}\sin(\epsilon \ln x)$

Here

$$r_{1,2} = \tfrac{1}{2}\{(1-\alpha) \pm [(\alpha-1)^2 - 4\gamma^2]^{1/2}\},$$
$$\delta = \tfrac{1}{2}(1-\alpha), \qquad \epsilon = \tfrac{1}{2}[4\gamma^2 - (\alpha-1)^2]^{1/2}.$$